

# Bulk fields with brane terms

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**Abstract.** In theories with branes, bulk fields get in general divergent corrections localized on these defects. Hence, the corresponding brane terms are renormalized and should be included in the effective theory from the very beginning. We review the phenomenology associated to brane kinetic terms for different spins and backgrounds, and point out that renormalization is required already at the classical level.

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## 1 Introduction

Models with extra dimensions and branes allow for new ways of addressing longstanding questions within the Standard Model (SM), such as the hierarchy problem or the structure of fermion masses and mixings. (See [1] for a review of their phenomenology.) These models are non-renormalizable and must be understood as effective theories valid at energies below a certain cutoff  $\Lambda$ . One can distinguish fields which propagate in all dimensions (“bulk fields”) from those which propagate only on the branes. Both kinds of fields can couple via operators which are necessarily localized on the branes [2, 3]. One can also consider brane localized operators involving only bulk fields. Since Poincaré invariance is broken by the presence of the brane (through brane fields or an orbifold projection), these operators typically receive divergent radiative corrections which require renormalization [4, 5]. Thus, their coefficients run and cannot be set to zero at all scales. It is therefore natural to include the brane terms from the very beginning in the (tree-level) effective action. In particular, one should expect the presence of Brane localized Kinetic Terms for all fields in the bulk (BKT). Their impact on phenomenology has been studied recently for different fields and backgrounds in a number of papers. It is our purpose here to review some of the main results in these works. We will also emphasize the breakdown of the low-energy expansion brought about by certain BKT, and comment on how to make sense of the theory.

## 2 Phenomenology of brane kinetic terms

The action for a bulk scalar in 5D with BKT has the form

$$S = \int d^4x \int dy (\partial_M \phi^\dagger \partial^M \phi + a_I^\phi \delta_I \partial_\mu \phi^\dagger \partial^\mu \phi + \dots), \quad (1)$$

where  $\delta_I \equiv \delta(y - y_I)$ ,  $y_I$  are the location of the branes and a sum over  $I$  is implicit. In the  $1/\Lambda$  expansion, the first term is order zero, whereas the second one is order  $1/\Lambda$ . In order for this expansion to be well defined, in the limit  $a_I^\phi \rightarrow 0$  the observable predictions should approach the ones for  $a_I^\phi = 0$ . However, this is not true for all BKT [6]. We will comment on “singular” BKT later in Sect. 3 and stick for the moment to BKT which behave smoothly when their coefficients go to zero. The phenomenological implications of these terms depend on the type of field, but they look pretty similar for different backgrounds (in compact scenarios). We discuss gauge fields, gravitons, and fermions in turn. We consider the coefficients of the BKT as free parameters of arbitrary size. Observe that even if they are small they can significantly modify the physics by breaking degeneracies among KK modes [7].

### 2.1 Gauge fields

BKT for gauge bosons in infinite extra dimensions have been considered in [8], with features analogous to the ones described for the graviton below. In this subsection we consider only bulk gauge bosons propagating in compact extra dimensions. The case of  $M_4 \times S^1/Z_2$  has been studied in [9]. The corresponding lagrangian is

$$\mathcal{L} = -\frac{1}{4} (\text{tr} F_{MN} F^{MN} + a_I^A \delta_I \text{tr} F_{\mu\nu} F^{\mu\nu}) + \dots, \quad (2)$$

with  $I = 0, \pi$ , and then the branes located at  $y_I = IR$  with  $R$  the orbifold radius. For  $a_0^A + a_\pi^A \leq -2\pi R$  there are ghosts, and for  $-2\pi R < a_I^A < 0$  tachyons, so we must take  $a_I^A \geq 0$ . From a 4D point of view, this theory is described by a Kaluza-Klein (KK) tower of spin 1 fields,

$A_\mu(x, y) = \sum_{n=0}^{\infty} \frac{f_n^A(y)}{\sqrt{2\pi R}} A_\mu^{(n)}(x)$ , with eigenfunctions  $f_n^A(y)$  of eigenvalue  $m_n$  diagonalizing (2). The BKT modify the diagonalization and normalization equations and, hence, the eigenfunctions and eigenvalues of the heavy modes. If only  $a_0$ , say, is nonvanishing, the latter decrease, whereas the former tend to decouple from the  $y_0$  brane. Hence, they are easier to reach at large colliders but, if matter is located at  $y_0$ , their production cross sections are smaller. See [9, 10] for the detailed dependence. For equal non-zero BKT at both branes, the lightest KK boson acts as a collective mode which becomes light and non decoupling for large  $a_I^A$ .

The integration of the KK modes gives four-fermion operators whose strengths are constrained by precision electroweak (LEP) data [11]. The reduction of the KK masses and couplings in the presence of BKT produces as a net result a relaxation of the corresponding limits, allowing for a lower compactification scale  $1/R$  [9, 10].

An analogous behaviour is shared by warped backgrounds. A detailed discussion of the implications of non-vanishing BKT for the Randall-Sundrum (RS) model [12] can be found in [13, 14], with similar conclusions. In particular, models with lower KK masses, maybe at the reach of next colliders, can be accommodated within experimental bounds.

Finally, we should mention that BKT must be kept small in GUT scenarios such as [15], as they can spoil gauge unification. On the other hand, they can be helpful in improving some features of models of electroweak symmetry breaking [16].

## 2.2 Graviton

Since gravity necessarily propagates in all the space, we must expect graviton BKT to be a generic feature of models with branes. These terms were first introduced to allow for an effective 4D Newton's law in 5D models with an infinite extra dimension [4]. The addition to the 5D curvature term  $R^{(5)}$  of a brane term  $a^g \delta_0 R^{(4)}$  gives rise to Newton's law  $V(r) \sim \frac{1}{r}$  at small distances  $r \ll a^g$ , and to a quadratic power law  $V(r) \sim \frac{1}{r^2}$  at larger ones  $r \gg a^g$ . Then, on phenomenological grounds the transition length  $a^g$  must be of the size of the present horizon. This modification of 4D gravity at large distances is very interesting since it allows for solutions of the cosmological constant problem, evading no-go theorems for theories which are 4D in the infrared [17, 18]. This model, however, presents strong quantum effects at distances  $(\frac{a^{g2}}{M_P})^{1/3} \sim 1000$  km, where  $M_P$  is the Planck mass, requiring new physics at this scale and then making the proposed model incomplete [19, 20]. (According to [21], however, this might be a calculational artifact.)

For a compactified extra dimension, the analysis of BKT is similar to the one of gauge bosons discussed above. The RS model with BKT,

$$\mathcal{L} = \frac{M_5^3}{4} \sqrt{-G} (R^{(5)} + a_I^g \delta_I R^{(4)} + \dots), \quad (3)$$

has been analysed in [22]. Again, the 4D KK gravitons may be significantly lighter for large  $a_I^g$  than for vanishing BKT, and their couplings to matter on the brane are smaller. They may avoid detection at large colliders as their widths may be too narrow to be observable. They also give rise to four-fermion operators when integrated out, but in this case their strengths are independent of  $a_I^g$  for a wide range of parameters. Finally, gravitational BKT are also useful in model building. For instance, they have been invoked in a model of gravity mediated supersymmetry breaking to avoid negative squared masses for scalars [23].

## 2.3 Fermions

The phenomenological consequences of BKT for bulk fermions in a 5D theory compactified on  $M_4 \times S^1/Z_2$  were presented in [24]. The lagrangian is

$$\mathcal{L} = (1 + a_I^L \delta_I) \bar{\Psi}_L i \gamma^\mu D_\mu \Psi_L + \dots \quad (4)$$

Again, the KK wave functions and masses are modified. As the new terms multiply the covariant derivative, the 4D gauge couplings for the different KK states get corrections both from the new wave functions and from the new effective couplings

$$g^{(mnr)} = \frac{g_5}{\sqrt{2\pi R}} \int dy (1 + a_I^L \delta_I) \frac{f_m^L f_n^L f_r^A}{2\pi R}, \quad (5)$$

where  $L, A$  label fermion and gauge boson wave functions, respectively, and  $g_5$  is the 5D gauge coupling. By gauge invariance, gauge bosons must propagate in the bulk if fermions do. Hence, the integration of the heavy KK gauge bosons give new contributions to four-fermion operators bounded by electroweak precision (LEP) data [11]. Using (5) and the corresponding experimental limits one can estimate the exclusion region for the compactification scale as a function of  $a_I^L$  [10, 24]. The constraints become stronger with growing BKT, in contrast with the situation in Sect. 2.1 and 2.2.

The observed fermions are zero modes which get their masses through Yukawa couplings after the spontaneous symmetry breaking of the SM. These couplings also generate mixings with the heavy KK fermions. Hence, a precise determination of the charged current mixing matrix constrains the mixing and masses of these exotic fermions, and thus the compactification scale [25]. In the presence of BKT their masses and mixings can be reduced, with the net effect of relaxing the experimental constraints. As a consequence there is room for producing new vector-like fermions at future colliders in many models [26].

## 3 General brane kinetic terms

The effects of the BKT in Sect. 2 are in general sizable only when the coefficients are large. Then, higher order terms in the effective theory may have to be taken into account.

In general, one must also include BKT with derivatives orthogonal to the brane. These have a nonanalytical behaviour, in the sense that they give big effects for arbitrarily small coefficients. A detailed discussion can be found in [6]. The most general kinetic terms for scalar, gauge bosons and fermions read

$$\mathcal{L} = (1 + a_I^\phi \delta_I) \partial_\mu \phi^\dagger \partial^\mu \phi - (1 + c_I^\phi \delta_I) \partial_y \phi^\dagger \partial_y \phi + \frac{b_I^\phi}{2} \delta_I (\phi^\dagger \partial_y^2 \phi + \partial_y^2 \phi^\dagger \phi), \quad (6)$$

$$\mathcal{L} = -\frac{1}{4} (1 + a_I^A \delta_I) F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (1 + c_I^A \delta_I) F_{5\nu} F^{5\nu}, \quad (7)$$

and

$$\mathcal{L} = (1 + a_I^L \delta_I) \bar{\Psi}_L i \not{\partial} \Psi_L + (1 + a_I^R \delta_I) \bar{\Psi}_R i \not{\partial} \Psi_R - (1 + b_I^\psi \delta_I) \bar{\Psi}_L \partial_y \Psi_R - b_I^\psi \delta_I (\partial_y \bar{\Psi}_R) \Psi_L + (1 + c_I^\psi \delta_I) \bar{\Psi}_R \partial_y \Psi_L + c_I^\psi \delta_I (\partial_y \bar{\Psi}_L) \Psi_R, \quad (8)$$

respectively. Let us discuss for illustration the scalar case in (6) with BKT at  $y_0$  only. For  $b_0^\phi \neq 0$ , the KK masses and wave functions diagonalizing the kinetic terms turn out to be independent of  $a_0^\phi$ ,  $b_0^\phi$  and  $c_0^\phi$ . Furthermore, their limits when  $b_0^\phi \rightarrow 0$  do not coincide with their values at  $b_0^\phi = 0$ , which in particular have a non-trivial dependence on  $a_0^\phi$ . This translates into dramatic changes in the observables for arbitrarily small  $b_0^\phi$ . This singular behaviour comes from the fact that we are considering branes of zero width. In perturbation theory, it manifests itself as singular products of delta functions at the origin. Taken at its faith value, this is catastrophic, as it means that all models in which these operators can be induced are badly defined as effective field theories. Fortunately, the singular behaviour can be smoothed down. The obvious way is to consider branes with finite thickness [27, 28]. However, predictions will then depend on the particular profile of the brane. Another possibility is to go beyond regularization of the brane, and renormalize the theory at the classical level through the introduction of higher order counterterms which cancel the singularities [6]. For instance, the second order counterterm

$$\mathcal{L}_{ct} = \frac{b_0^{\phi^2}}{4} \left\{ -\delta_0^2(y) \partial_\mu \phi^\dagger \partial^\mu \phi - \partial_y [\delta_0(y) \phi^\dagger] \partial_y [\delta_0(y) \phi] + \delta_0^2(y) (\phi^\dagger \partial_y^2 \phi + \partial_y^2 \phi^\dagger \phi) \right\}, \quad (9)$$

makes the limit  $b_0^\phi \rightarrow 0$  well-defined to second order. At this order, the effect of the singular BKT can be absorbed into a redefinition of the coefficients of the nonsingular BKT. It is plausible that, after proper renormalization, the effects of all possible BKT can be parametrized only by the coefficients of the nonsingular ones. If this is the case, their phenomenology will reduce to the one discussed above.

To finish, let us mention that the situation in supersymmetric theories is similar, except for the fact that some of the possible BKT are stable against radiative corrections [6] (see also [29, 30]).

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